

# Massive Degeneracy and Goldstone Bosons: A Challenge for the Light Cone\* <sup>†</sup>

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Wherein it is argued that the light front formalism has problems dealing with Goldstone symmetries. It is further argued that the notion that in hadron condensates can explain Goldstone phenomena is false.

## I. A DEFINITION

You might well ask why I have decided to talk about the meaning of the Goldstone theorem and physics on the Light Front. After all, while important, this is not really a new subject. The answer is simple. I have chosen to give this talk, rather than talk about new research, because at past Light Cone meetings I have discovered that this subject is not as well understood by all of the participants as it should be. Therefore, people have not understood the discussions that have taken place between Stan Brodsky and me on the question of *in hadron condensates* and the problems with the uniqueness of the Light Front vacuum state. I have titled this talk "Massive Degeneracy and Goldstone Bosons: A Challenge for the Light Cone" to emphasize the importance of this issue for a proper understanding of PCAC and Current Algebra.

As it turns out, during my recent travels I found that my titles are not always easily understood. So I went back to the Wikipedia for a definition of *degeneracy*. This is what I found:

- de-gen-er-a-cy [ di?jnn?r?ssee ] noun (plural de-gen-er-a-cies)

### Definition:

1. bad behavior: immoral, depraved, or corrupt behavior, or an instance of this
2. worsened condition: a condition that is worse than normal or worse than before
3. worsening of condition: the process of becoming physically, morally, or mentally worse
4. quantum physics states of equal energy: the condition of two or more quantum states having the same energy.

I would love to discuss the first three topics, but alas, I will be discussing the fourth in this talk. The point that I want to make in the time allotted to me, is that the successes of PCAC and Current Algebra require us to conclude that in our world the hadron sector is very close to a theory where  $SU(3) \times SU(3)$  is realized as a Goldstone (other people say spontaneously broken) symmetry. A corollary of this, is that in the limit of an exact Goldstone symmetry, the ground state of the theory is enormously degenerate and this feature of QCD is not apparent in the Light Front formulation of the theory. *This is the feature of QCD that has to be better understood.*

## II. A PARABLE

What would a general talk be without a parable ? Nothing! So, having said that, I will begin with a modified version of a parable that I believe I first heard from Sydney Coleman.

Once upon a time in a universe far, far smaller than ours there lived the famous savant Doctorus E, who was an expert on practically everything and worked at the famous Crystallus U. One day as the Doctorus was deep in

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thought a student interrupted with a strange observation. He said, "Doctorus, I have just discovered an amazing thing! The world is translation invariant!" Never at a loss for a response the famous savant replied "Dummkopff, everybody knows that! Come here and look out the hyper-viewer at what can be seen in the sky. Obviously if you

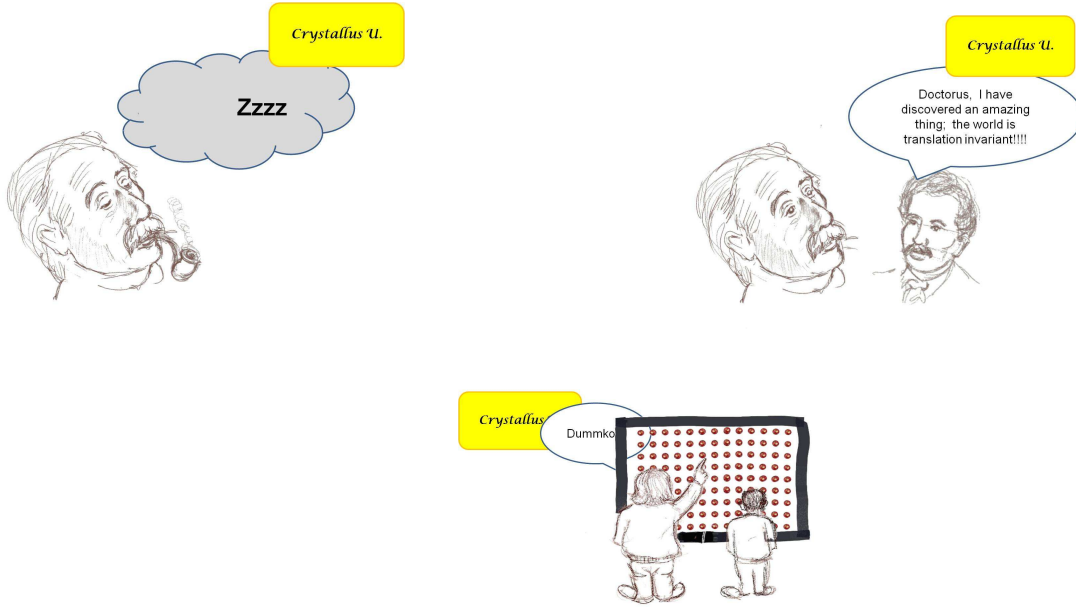


FIG. 1: Read the pictures left to right and top to bottom

move the entire world over by one grelp (an astronomical unit in Crystallus) everything looks the same. We have known that forever".

"But no Doctorus", said the student, "that is not what I meant! I mean that the laws of physics don't change even if we translate the world by an arbitrarily small amount". To which the savant responded "How could you know that? It would take an infinite amount of energy to move the whole world by an arbitrarily small amount! How could we test this idea?"

"That's the neat thing Doctorus", said the student. You don't have to move the whole world to find out if there is, what I call, a *hidden symmetry*. All you have to do is see how much energy it takes to excite an arbitrarily long wavelength excitation. The consequence of the hidden symmetry is that this energy will go to zero as the wavelength goes to infinity. In addition, these long wavelength excitations satisfy sum rules constraining their interactions with imperfections"

"Hmm", said the savant, "let me think about this."

As it turned out the young student was correct. In fact, parable aside, translation invariance (by arbitrarily small translations) is realized as a *hidden symmetry* in a crystal. The excitations are called *phonons* and these phonons have no mass (i.e., their energy as a function of momentum goes to zero as the momentum vanishes).

### III. EXAMPLES OF GOLDSTONE SYMMETRIES

Of course, I wouldn't be talking about this if the only example of a hidden symmetry was phonons in a crystal. In fact, condensed matter physics is replete with examples of systems that exhibit this phenomenon.

For example, ferromagnets are objects that exhibit spontaneous magnetization; i.e., in these systems magnetic moments of atoms align with one another to produce an observable, persistent magnetic field. Since this magnetic field points in a definite direction, it follows that the rotational invariance of the full theory is *hidden* from us. The massless excitations (or gapless excitations of this system are called *magnons*). Since the magnetization could point in any direction, it follows that the ground state of this system is infinitely degenerate. Similarly, there are anti-ferromagnets, where the spins anti-align, but point in a definite direction. (High temperature superconductors exhibit this behavior when they are under-doped.) Once again the hidden symmetry is rotational invariance and the massless excitations are called *spin-waves*.

Finally, not to be outdone by the world of condensed matter physics, particle physics also has its share of hidden (or Goldstone) symmetries. For the purposes of this discussion I will avoid theories that have the additional complication



FIG. 2: Both the ferromagnet and anti-ferromagnet have rotational invariance as a hidden symmetry.

of the Higgs phenomenon taking place and will concentrate on QCD. It is an old story that the only consistent explanation of the Goldberger-Treiman relation, Adler-Weissberger calculation of the  $g_A/g_V$  sum-rule, the sum rule for the squares of the masses of the pseudoscalar mesons, Weinberg's formula for the  $\pi - \pi$  scattering lengths, Dashen-Weinstein theorem on the slope of the form factors in  $Kl_3$  decay, etc., is that QCD is close to a theory in which  $SU(2) \times SU(2)$  (and in fact  $SU(3) \times SU(3)$ ) is an exact but *hidden*, (or Goldstone) symmetry. Moreover, the only source of symmetry breaking are the quark mass terms in the Hamiltonian. In other words, if quark masses were zero, then this symmetry would be exact and the  $\pi$ ,  $K$  and  $\eta$  meson would all have zero mass. They would be, like the phonons, magnons or spin-waves, the massless excitations associated with the hidden symmetry. By the way, one reason I like calling this sort of symmetry a hidden or Goldstone symmetry, rather than the more popular *spontaneously broken* symmetry is that it avoids having to talk about a "really broken, spontaneously broken, symmetry" when quark masses are non-zero.

The key point I want to make in the rest of this talk is that it is generally true, as is obvious in the case of the condensed matter examples I talked about, that in order for there to be Goldstone bosons (these massless excitations) there has to be something to *wiggle*.

#### IV. FORMALITIES

Since I see friends in the audience with a bit of a formal bent, I will spend a few moments giving some formal insight into the meaning of the Goldstone theorem and what it says about there having to be stuff that wiggles. A more extensive discussion of some of these issues can be found in my Heidelberg lectures[1].

The story begins with that perennial favorite, Noether's theorem. You all know the theorem, you learned it in grade school. It says, that if a Lagrangian has a continuous symmetry then there exists a locally conserved current associated with that symmetry; i.e.,

$$\partial_\mu j^\mu = 0. \quad (1)$$

The usual follow up to the proof of this theorem is the observation that as a consequence of current conservation there is an associated conserved charge

$$Q = \int d^3x j^0(x). \quad (2)$$

It is usually argued that a consequence of the current conservation equation that the time derivative of this charge vanishes

$$\partial_0 Q = \int d^3x \partial_0 j^0(x) = - \int d^3x \vec{\nabla} \cdot \vec{j}(\vec{x}) = 0, \quad (3)$$

at least if surface terms can be neglected. Then, so the story goes, we have a time independent Hermitian operator that can be exponentiated to provide a unitary representation of the symmetry group. It is here that the story becomes more complicated. As with all things in physics there is often a *gotcha*.

To better understand what the gotcha is, define the operator

$$Q_\Omega = \int_\Omega d^3x j^0(x), \quad (4)$$

where the integration is over a finite three volume  $\Omega$ . The good thing about his operator is that it exists and the local nature of the commutator of the current with local fields guarantees that

$$\lim_{\Omega \rightarrow \infty} [Q_\Omega, \phi(\vec{x})] \quad (5)$$

exists, since once  $\Omega$  is larger than the intersection of either the past or future light cone of the point  $\vec{x}$  with the surface of integration in Eq.5. This observation tells us that by taking the limit  $\Omega \rightarrow \infty$  in all commutators of  $Q_\Omega$  with all local observables we obtain an automorphism of the space of local observables: the question is whether this is an *inner automorphism*? In other words, is there a Hermitian operator  $Q$  defined on the Hilbert space that generates the same automorphism. If so, it can be exponentiated. Furthermore, if the conserved currents close to the algebra of a compact Lie group, then so will the conserved Hermitian charges and therefore they will generate a unitary representation of the Lie group on the space of physical states. In that case we say that the conserved currents are realized as a *Wigner symmetry*. The hallmark of a Wigner symmetry is that the states are grouped into finite dimensional representations of the Lie group (since it is compact) and they all have the same mass; i.e., they are degenerate. Also, using the Wigner-Eckart theorem, we are able to relate matrix elements of operators that transform as irreducible representations of the group, to one another. Hence, we get relations between coupling constants, transition matrix elements, etc.

Suppose, however, that the automorphism defined by the conserved currents is not inner? This can only be the case if the limit of the operator  $Q_\Omega$  fails to exist as  $\Omega \rightarrow \infty$ . This will happen if there is a massless particle coupled by the current to the vacuum state. When this happens the existence of the conserved currents implies that the system has a non-trivial symmetry, but this symmetry is no longer realized by having the states of the theory bundled into nice finite dimensional irreducible representations of the Lie group. Rather, the consequences of the symmetry are exact low-energy theorems controlling the low energy behavior of the massless particle that is coupled by the conserved currents to the vacuum. Such a symmetry is not immediately obvious to us and for that reason I will adopt Coleman's terminology and refer to it as a *hidden* or *Goldstone* symmetry. This is the sort of symmetry realized by the conserved axial vector currents in QCD in the limit of vanishing quark masses. The Goldstone bosons, i.e. the massless particles coupled to the vacuum state by these currents, are the  $\pi$ ,  $K$  and  $\eta$  mesons. Some of the consequences of this symmetry are: the Goldberger-Treiman relation, the Adler-Weissberger relation, the PCAC self-consistency conditions, the Dashen-Weinstein theorem on the form factors in  $K l_3$  decay, and Weinberg's theorem on the behavior of low energy  $\pi - \pi$  scattering. This list is by no means complete, I give it only to convince you that there is a great deal of experimental evidence that points to the fact that the axial current part of chiral  $SU(3) \times SU(3)$  is realized as a hidden or Goldstone symmetry, whereas the symmetry generated by the vector currents is of the Wigner type.

Of course, since this talk is about massive degeneracies, I should explicitly point out that the existence of the massless particle created by this current means that the lowest energy state of the theory is enormously degenerate. This is because we can add any number of zero momentum massless particles to the vacuum without increasing the energy. It is the fact that *the light-front formalism insists that the vacuum is the unique lowest energy state* that makes reconciling the light front treatment of QCD with the real world so problematic. I would also like to point out that any argument that says *in hadron condensates* can explain the Goldstone boson phenomenon is simply incorrect, in that it cannot explain this huge vacuum degeneracy.

## A. Finite Volume Considerations

I now have to say a few words about what happens if I make the spatial volume finite, instead of infinite. Why do I feel compelled to do this? Because all non-perturbative approaches to dealing with QCD involve beginning with a system in a finite volume and then taking the volume to infinity, and when the volume is finite, the ground state of the theory is unique. Where then is the massive degeneracy I spoke of?

To clarify this issue, let us return to the example of the ferromagnet. Imagine we make a state that is the tensor product of essentially the same norm one spin state on each lattice site, and assume that this state is constructed so that the expectation value of the spin (or magnetization) points in a definite direction. This product state is a contender for the infinite volume magnetized state.

If one now applies a rotation to the spins on each site one obtains a new state for which the magnetization points in another direction. Now, because the volume is finite the overlap of two such states is given by

$$|\Psi\rangle = \prod_i^N |\psi_i\rangle \quad \text{and} \quad |\Phi\rangle = \prod_i^N |\phi_i\rangle \quad (6)$$

$$\langle \Psi | \Phi \rangle = \prod_i^N \langle \psi_i | \phi_i \rangle. \quad (7)$$

Note, since two non-aligned states of unit length have an overlap whose magnitude is less than one; i.e.,  $\|\langle\psi_i|\phi_i\rangle\| < 1$ , it follows that the overlap of the finite volume states,  $\Psi$  and  $\Phi$  goes to zero exponentially as that number to the power  $N$ . Furthermore, for a spin-spin Hamiltonian it is clear that

$$\langle\Psi|H|\Psi\rangle = \langle\Phi|H|\Phi\rangle, \quad (8)$$

$$\langle\Psi|H|\Phi\rangle \approx X^N \rightarrow 0, \quad (9)$$

where  $X$  is some number less than unity. Since the different states are not orthogonal (but they are unit length) one can use them to form an orthonormal basis and diagonalize the Hamiltonian truncated to this space of states. These will be the correct lowest lying eigenstates in the limit of large  $N$  and they will be split by an amount that goes to zero as  $N \rightarrow \infty$ . It is the fact that these states become split by exponentially small amounts as the number of sites gets large that explains how a finite size ferromagnet seems to form. Clearly, the state in which the ferromagnet points in a definite direction is a linear combination of the eigenstates we constructed. Since the splitting between these states is so small, turning on a small magnetic field will put the system into this *magnetized* state. If this field is then turned off adiabatically the different eigenstates will evolve in time with slightly different phases, due to their energy differences. However since these energy differences are exponentially small, it will take an exponentially long time to see the magnetization vanish.

The key point of all of this, is not why we can see ferromagnets that have a finite volume. Rather it is that the signal of the infinite degeneracy of the infinite volume limit, is an enormous number of nearly degenerate states whose number grows rapidly with increasing volume. This enormous degeneracy is what a light front calculation, done in finite volume, should see. This is what, to the best of my knowledge, isn't apparent yet. The statement that the virtue of the light front is that the vacuum state is empty (and unique) is, in the case of spontaneous symmetry breaking, a problem, not a virtue.

## V. WHAT HAPPENS IN THE INSTANT FORMALISM ?

Having criticized the light front approach because it doesn't make it easy to address Goldstone symmetries, it would be remiss of me to not argue that this problem is less difficult in the instant formalism. I will now contend that for a non-Abelian gauge theory, such as QCD, the formation of Goldstone bosons is an inescapable property of the strong coupling limit of the theory.

### A. The Schwinger Model

Given the limitations of time, I will begin with a very short discussion of the 1 + 1-dimensional Schwinger model, because it exhibits most of the physics I wish to discuss. After that I will make an even briefer foray into QCD. The point of this, as I have already said, is to show that for these theories it is trivial to argue that the strong coupling limit of the theory explains why the vacuum state can be very degenerate and support the existence of Goldstone bosons.

We begin with the formulation of the lattice version of the Schwinger model in  $A_0 = 0$  gauge. The setting for the model is a 1-dimensional lattice whose sites are labelled by the integer  $j$ . The fermions in this model live on the sites and are represented by the two-component fermion field  $\psi_j$  and its conjugate  $\psi_j^\dagger$ . The Abelian gauge field of the model,  $A_j$ , lives on the link joining the pair of lattice sites  $j$  and  $j + 1$ . The conjugate variable to  $A_j$  is the electric field variable  $E_j = \dot{A}_j$  and they satisfy the canonical commutation relations  $[A_j, E_{j'}] = i\delta_{j,j'}$ . Since the variable  $A_0$  appearing in the Schwinger model Lagrangian, it follows that the Maxwell equation coming from varying the Lagrangian with respect to  $A_0$  will not be an equation of motion. With these definitions, if we follow the usual prescription, we construct the Hamiltonian of the generic form of the lattice Schwinger model:

$$H = H_E + H_f, \quad (10)$$

where

$$\begin{aligned} H_E &= \frac{g^2}{2} \sum_n E_n^2 \\ H_f &= \sum_{n,n'} (\psi_n^\dagger)^\alpha K(n - n')_{\alpha\beta} e^{-i \sum_{j=n}^{n'-1} A_j} (\psi_{n'})^\beta, \end{aligned} \quad (11)$$

where the kinetic term  $K(n - n')_{\alpha\beta}$  is a two-by-two matrix for each value of  $n - n'$ , the fermion fields satisfy the anti-commutation relations

$$\{(\psi_n^\dagger)^\alpha, (\psi_{n'})^\beta\} = \delta_{n,n'} \delta_{\alpha,\beta}. \quad (12)$$

The link fields satisfy the harmonic oscillator commutation relations given above.

Now in any number of dimensions, the missing Maxwell equation is just the Gauss law. In one dimension this law takes the particularly simple and suggestive form

$$G_j = E_{j+1} - E_j - \psi_j^\dagger \psi_j = 0. \quad (13)$$

The important fact is that although this equation is not one of the Euler-Lagrange, or Heisenberg equations of the theory, it follows from the specific form of the Hamiltonian that all of the operators  $G_j$  commute with the Hamiltonian; i.e.,

$$[G_j, H] = 0 \quad \forall j. \quad (14)$$

From this it follows that, although the theory contains states that do not satisfy the Gauss law, we are free to restrict ourselves to the subspace of states which do, since the Hamiltonian will never take us out of this subspace. Furthermore, since the operators  $G_j$  are precisely the operators that generate local time-like gauge-transformations, all gauge invariant operators must also commute with the  $G_j$ 's. From this discussion we see that by beginning in  $A_0 = 0$  gauge we over quantize the theory, in that canonical manipulations can create more states than we wish; however, thanks to gauge invariance we can select a subspace of states that gives us the theory we are interested in.

Obviously, since we have the choice of how to choose our basis states, we see that satisfying the Gauss law will be most easily done if we work in a basis in which the operators  $E_j$  and  $\rho_j = \psi_j^\dagger \psi_j$  are diagonal. Since we are dealing with fermions, we have the possibility of having four possible fermion states: these correspond to the state having no particles, one particle of charge  $-1$ , or one anti-particle of charge  $+1$ , or finally, one particle and one anti-particle on a single lattice site. Note, if we impose the Gauss condition then specifying the charges on each site completely (up to a constant background field which we will take to be zero) specifies the state. Note, since the Hamiltonian only contains the operators  $e^{iA_j}$ , it can only couple together states in which the electric field changes in absolute value by one unit.

At this juncture I am in a position to keep my promise and argue that in the limit  $g^2 \rightarrow \infty$  the Schwinger model has an infinitely degenerate ground-state. Moreover, I will argue that for very large, but finite  $g^2$  the theory has a Goldstone boson that, due to the anomaly, fails to appear as we take the limit  $g^2 \rightarrow 0$ .

I begin by noting the the term  $\frac{g^2}{2} \sum_j E_j^2$  means that when  $g^2 \gg 1$  having any flux (i.e. any non-vanishing value for  $E_j$ ) costs a lot of energy. Thus, the ground state of the theory in this limit must be a state where  $E_j = 0$  for all links. By the Gauss condition this means that the charge on each site must vanish. However, we have already seen that there are two possible zero charge states for each site. Thus we see that in the large  $g^2$  limit, for a lattice with  $V$  sites, there will be  $2^V$  degenerate states with energy zero. All other states will have infinite energy. Now, if we take  $g^2$  large but finite, then we observe that the kinetic term can separate a pair and create a particle and anti-particle on different sites joined by a unit of flux. Since this is a high-energy state the energy denominator appearing in perturbation theory is large and so we are invited to treat the effects of the kinetic term on the ground state by second order perturbation theory. However, since the ground is so degenerate, we must do degenerate perturbation theory, since the different degenerate states are mixed by the kinetic term. This leads us to an effective Hamiltonian which is our friend the Heisenberg anti-ferromagnet, and as I already said, this theory has a symmetry that is realized in the Goldstone mode. Of course, there are no little magnetic spins in this case, rather the role of spin has to do with the chiral charge of the state which, for each site, is the sum of the particle and anti-particle number on that site minus one.

Time doesn't permit me to discuss this model further, especially the interesting story of what happens to the Goldstone mode in the continuum (i.e.  $g^2 \rightarrow 0$ ) limit. Since what happens in this case is specific to the anomaly in the axial current, it presumably is not relevant to the case of the octet of axial currents in QCD. I refer you to my paper with Kirill Melnikov[2] on the subject for all of the details.

## B. What About QCD ?

At this point I can only give the briefest summary of what happens in QCD. If one works in the corresponding version of  $A_0 = 0$  gauge, the story parallels that of the Schwinger model. Once again, gauge invariance requires that in the large coupling limit the color charge on every site must vanish in order to avoid having non-vanishing flux on

any link. Thus, each site can have as many  $q\bar{q}$  or  $qqq$  states as are allowed by the exclusion principle. If we simply focus on the possible mesons that there are a large number of zero energy states on each site. As in the case of the Schwinger model we do degenerate second order perturbation theory to understand what happens when we turn on the kinetic terms.

The result is that we obtain a frustrated  $SU(12) \times SU(12)$  anti-ferromagnet. The frustration is due to the presence of next nearest neighbor hopping terms. The same terms break the global symmetry to chiral  $SU(3) \times SU(3)$ . It is straightforward to show that, in this system, the vector  $SU(3)$  symmetry is realized in Wigner fashion, meaning that there are degenerate  $SU(3)$  multiplets of particles with non-vanishing mass, but the axial part of the symmetry is realized in Goldstone mode. The massless multiplet of mesons are the  $\pi$ ,  $K$  and  $\eta$  mesons. Another bonus is that in this limit we get the good predictions of the ratio of magnetic moments obtained in the old  $SU(6)$  symmetry scheme, but none of the bad predictions. A complete discussion of this approach appears in paper by myself, Sid Drell, Helen Quinn and Ben Svetitsky[3]-[4].

## VI. THE CHALLENGE

This talk can be summarized as follows:

- Exact symmetries can be realized in Wigner or Goldstone mode.
- When a symmetry is realized in Wigner mode the states of the theory form degenerate irreducible representations of the symmetry group and the lowest energy state is unique.
- When a symmetry is realized in Goldstone mode the lowest energy state of the theory is infinitely degenerate, the states of the theory do not form irreducible representations of the symmetry group and there are massless particles coupled by the conserved currents to any one of the possible ground states.
- In finite volume the signal of a Goldstone realization of a symmetry is that the number of nearly degenerate states grows rapidly with increasing volume and the gap between these states shrinks exponentially with the volume.
- The existence of a *condensate* such as the magnetization, for a ferromagnet, or the staggered magnetization for an anti-ferromagnet, signals a Goldstone symmetry. This is because this condensate transforms non-trivially under the symmetry transformations and so its existence implies the ground state isn't unique.
- PCAC means that the pion, kaon and eta are would be Goldstone bosons of the theory where the quark masses are set to zero. This interpretation is overwhelmingly supported by experimental data. This means that these particles are really the wiggling of the order parameter or condensate.
- Finally, in order for the Goldstone particle to exist there has to be something to wiggle every place where the particle can exist. *This means that the condensate that is the order parameter for this Goldstone symmetry cannot be confined to the interior of hadrons.*

Thus, to reiterate, the challenge for the Light Front is to show how the formalism gives rise to this sort of pattern of degeneracy when the physical volume of space becomes large.

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